Differential Equations

Question1

If $y = At^2 + \frac{B}{t}$ (A, B are parameters) is general solution of the differential equation $f(t)y''(t) + g(t)y'(t) + h(t)y = 0$ then $2f(t) + t^2h(t) =$

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Options:

A.

$g(t) - h(t)$

B.

$g(t) + f(t)$

C.

$g(t)f(t)$

D.

$(f(t))^{g(t)}$

Answer: C

Solution:

$\because y = At^2 + \frac{B}{t}$

$\Rightarrow y' = 2At - Bt^{-2}$ and $y'' = 2A + 2Bt^{-3}$

Substituting into the differential equation

$$f(t)\left(2A + 2Bt^{-3}\right) + g(t)\left(2At - Bt^{-2}\right) + h(t)\left(At^2 + Bt^{-1}\right) = 0$$
$$\Rightarrow A\left[2f(t) + 2tg(t) + t^2h(t)\right] + B\left[2t^{-3}f(t) - t^{-2}g(t) + t^{-1}h(t)\right] = 0$$

the coefficients must be zero.

So, $2f(t)+2tg(t)+t^2h(t)=0 \quad \ldots(i)$

and $2t^{-3}f(t)-t^{-2}g(t)+t^{-1}h(t)=0 \quad \ldots(ii)$

Multiply Eq. (ii) by t^3

$2f(t)-tg(t)+t^2h(t)=0 \quad \ldots(iii)$

Subtracting Eqs. (i) and (ii), we get

$$[2f(t)+2tg(t)+t^2h(t)]$$
$$-[2f(t)-tg(t)+t^2h(t)]=0 \Rightarrow 3tg(t)=0$$

$\because t \neq 0$, so put $g(t)=0$ in Eq. (i)

$2f(t)+t^2h(t)=0$

$\because \quad g(t)=0$, so $g(t)\cdot f(t)=0$

Hence, $2f(t)+t^2h(t)=g(t)\cdot f(t)$

Question2

The general solution of the differential equation $(2x-y)^2dy-2(2x-y)^2dx-2dx=0$ is

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Options:

A.

$\log(2x-y)=2x+C$

B.

$(2x-y)^3+4y=C$

C.

$(2x-y)^3+6x=C$

D.

$\log(2x-y)=2y+C$

Answer: C

Solution:

$\because (2x-y)^2 dy - 2(2x-y)^2 dx - 2dx = 0$

put $v = 2x - y \Rightarrow dv = 2dx - dy$

$\Rightarrow dy = 2dx - dv$

So, $v^2(2dx - dv) - 2v^2 dx - 2dx = 0$

$\Rightarrow \quad 2v^2 dx - v^2 dv - 2v^2 dx - 2dx = 0$

$\Rightarrow -v^2 dv - 2dx = 0 \Rightarrow v^2 dv + 2dx = 0$

$\Rightarrow \int v^2 \cdot dv = -2 \int dx \Rightarrow \frac{v^3}{3} = -2x + c$

$\Rightarrow \quad \frac{(2x-y)^3}{3} = -2x + c$

$\Rightarrow \quad (2x-y)^3 = -6x + 3c$

$\Rightarrow \quad (2x-y)^3 + 6x = C$, where $C = 3x$

Question3

The general solutions of the differential equation $x \log x dy = (x \log x - y) dx$ is

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Options:

A.

$(x-y)\log x + x = C$

B.

$x - y = \frac{x}{\log x} + C$

C.

$y - x = \frac{x}{\log x} + C$

D.

$(y-x)\log x+x=C$

Answer: D

Solution:

$x\log x dy=(x\log x-y)dx$

$$\begin{aligned} &\Rightarrow \quad x\log x\frac{dy}{dx}=x\log x-y \\ &\Rightarrow \quad \frac{dy}{dx}+\frac{y}{x\log x}=1 \end{aligned}$$

Comparing with $\frac{dy}{dx}+P(x)\cdot y=Q(x)$

$\therefore \text{IF}=e^{\int \frac{1}{x\log x}dx}=e^{\log|\log x|}=\log x$

Multiply IF both sides

$$\begin{aligned} &\quad \log x\cdot\frac{dy}{dx}+\frac{y}{x}=\log x \\ &\Rightarrow \quad \frac{d}{dx}(y\log x)=\log x \\ &\Rightarrow \quad y\log x=\int \log x\cdot dx \\ &\Rightarrow \quad y\log x=x\log x-x+C \\ &\Rightarrow \quad \log x(y-x)+x=C \end{aligned}$$

Question4

If a and b are arbitrary constants, then the differential equation corresponding to the family of curves $y=\tan(ax+b)$ is

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Options:

A.

$(1+x^2)y_2-2yy_1+y=0$

B.

$(1+y^2)y_2-2yy_1^2=0$

C.

$(1+x^2)y_2 + 2yy_1^2 = 0$

D.

$(1+y^2)y_2 - 2yy_1^2 + y = 0$

Answer: B

Solution:

Given,

$$y = \tan(ax+b)$$
$$\Rightarrow \tan^{-1} y = ax + b$$
$$\Rightarrow \frac{1}{1+y^2}\frac{dy}{dx} = a$$
$$\Rightarrow \frac{1}{(1+y^2)}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{-1}{(1+y^2)^2} \cdot 2y\frac{dy}{dx} = 0$$
$$\Rightarrow (1+y^2)\frac{d^2y}{dx^2} - 2y\left(\frac{dy}{dx}\right)^2 = 0$$
$$\Rightarrow (1+y^2)y_2 - 2yy_1^2 = 0$$

Question5

The general solution of the differential equation $xy(y+2)dy + (y^3-1)dx = 0$ is

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Options:

A.

$\log|x+2y| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{y-x}{\sqrt{3}x}\right) = C$

B.

$\log|2x-y| + \frac{2}{3}\tan^{-1}\left(\frac{x-y}{\sqrt{3}x}\right) = C$

C.

$\log |xy-x|+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=C$

D.

$\log |x+y|+\frac{2}{3}\tan^{-1}\left(\frac{x-2y}{\sqrt{3}x}\right)=C$

Answer: C

Solution:

Given, $xy(y+2)dy+\left(y^3-1\right)dx=0$

$$\left(y^3-1\right)\frac{dx}{dy}-xy(y+2)=0$$
$$\Rightarrow \frac{dx}{dy}=\frac{xy(y+2)}{y^3-1}$$
$$\Rightarrow -\frac{dx}{x}=\frac{y(y+2)}{y^3-1}dy$$
$$\Rightarrow -\int\frac{dx}{x}=\int\frac{y(y+2)}{(y-1)\left(y^2+y+1\right)}dy$$

Now, $\frac{y(y+2)}{(y-1)\left(y^2+y+1\right)}=\frac{A}{y-1}+\frac{B(2y+1)+C}{y^2+y+1}$

$$\Rightarrow y^2+2y=A\left(y^2+y+1\right)+B(y-1)(2y+1)+C(y-1)$$

By comparing coefficients

$1=A+2B|2=A-B+C|0=A-B-C$

Solving, we get

$$2=2C\Rightarrow C=1$$
$$A+2B=1$$
$$A-B=1$$
$$-\ \ +\ \ -$$
$$B=0\Rightarrow A=1$$
$$\Rightarrow -\ln x=\int\frac{1}{y-1}+\frac{1}{y^2+y+1}$$
$$=\int\left(\frac{1}{y-1}+\frac{1}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)dy$$
$$\Rightarrow -\ln x=\ln(y-1)+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)+C$$
$$\Rightarrow C=\ln|x(y-1)|+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)$$
$$\Rightarrow \ln|xy-x|+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=C$$

Question6

The general solution of the differential equation $(1+\sin^2 x)\frac{dy}{dx}+y\sin 2x=\cos x+\sin^2 x\cos x$ is

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Options:

A.

$(\sin 2x)y=\sin^2 x+C$

B.

$(1+\sin^2 x)y=\sin x-\frac{\sin^3 x}{3}+C$

C.

$(1+\sin^2 x)y=\sin x+\frac{\sin^3 x}{3}+C$

D.

$(\sin 2x)y=\sin x+\sin^2 x+C$

Answer: C

Solution:

We have,

$$(1+\sin^2 x)\frac{dy}{dx}+(\sin 2x)y=\cos x+\sin^2 x\cos x$$

$$\Rightarrow \frac{dy}{dx}+\left(\frac{\sin 2x}{1+\sin^2 x}\right)y=\frac{\cos x\left(1+\sin^2 x\right)}{1+\sin^2 x}$$

$$\Rightarrow \frac{dy}{dx}+\left(\frac{\sin 2x}{1+\sin^2 x}\right)y=\cos x$$

$$\therefore \quad P=\frac{\sin 2x}{1+\sin^2 x}, Q=\cos x$$

$$\therefore \text{IF} = e^{\int P dx}=e^{\int \frac{2\sin x\cos x}{1+\sin^2 x}dx}$$

$$=e^{\ln\left|1+\sin^2 x\right|}$$

$$=1+\sin^2 x$$

∴ Solution of differential equation

$$y(\text{ IF }) = \int Q(\text{ IF })dx + C$$

$$\Rightarrow y\left(1+\sin^2 x\right)$$

$$= \int \cos x\left(1+\sin^2 x\right)dx$$

$$= \int \cos x dx + \int \cos x \sin^2 x dx$$

$$y\left(1+\sin^2 x\right) = \sin x + \frac{\sin^3 x}{3} + C$$

Question7

If the slope of the tangent drawn at any point (x, y) on a curve is $(x+y)$, then the equation of that curve is

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Options:

A.

$y = ce^x + 1 + x$

B.

$y = ce^x - x$

C.

$y = ce^{-x} - 1 - x$

D.

$y = ce^x - 1 - x$

Answer: D

Solution:

Since, the slope of tangent to a curve at any point (x, y) is $x+y$.

$\therefore \quad \frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$

This is a linear differentiation equation

So, $P = -1$ and $Q = x$

Now, IF $= e^{\int P dx} = e^{\int -dx} = e^{-x}$

$$\Rightarrow \quad e^{-x} \cdot \frac{dy}{dx} - e^{-x} y = x e^{-x}$$
$$\Rightarrow \quad \frac{dy}{dx}\left(y \cdot e^{-x}\right) = x e^{-x}$$

Integrating both sides, we get

$$\begin{aligned} y \cdot e^{-x} &= \int x e^{-x} dx \\ &= -x e^{-x} - \int -e^{-x} dx \\ &= -x e^{-x} + \left(-e^{-x}\right) + c \end{aligned}$$

where $C =$ constant of integration $\Rightarrow y e^{-x} = -x e^{-x} - e^{-x} + c$

$\Rightarrow y = -x - 1 + c e^{x}$, where c is arbitrary constant.

Question8

The solution of the differential equation $x^2(y+1)\frac{dy}{dx} + y^2(x+1)^2 = 0$, when $y(1) = 2$, is

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Options:

A.

$\log\left|x^2 y\right| = \frac{2}{x} + \frac{1}{y} + x - 1$

B.

$\log\left|\frac{1}{4} x^2 y\right| = \frac{1}{x} + \frac{2}{y} + x - 1$

C.

$\log\left|\frac{1}{2} x^2 y\right| = \frac{1}{x} + \frac{1}{y} - x - \frac{1}{2}$

D.

$\log\left|\frac{1}{3} x^2 y\right| = \frac{1}{x} + \frac{1}{y} - x + \frac{1}{2}$

Answer: C

Solution:

Given, differential equation is

$$x^2(y+1)\frac{dy}{dx}+y^2(x+1)^2=0, y(1)=2$$
$$\Rightarrow \quad x^2(y+1)\frac{dy}{dx}=-y^2(x+1)^2$$
$$\Rightarrow \quad \frac{(y+1)}{y^2}dy=-\frac{(x+1)^2}{x^2}dx$$

Integrating to both sides, we get

$$\int\frac{(y+1)}{y^2}dy=-\int\frac{(x+1)^2}{x^2}dx$$
$$\Rightarrow \quad \int\left(\frac{1}{y}+\frac{1}{y^2}\right)dy$$
$$=-\int\left(1+\frac{2}{x}+\frac{1}{x^2}\right)dx$$
$$\Rightarrow \ln|y|-\frac{1}{y}=-\left(x+2\ln|x|-\frac{1}{x}\right)+C,$$

where $C=$ constant

$$\Rightarrow \quad \ln|y|+2\ln|x|=\frac{1}{y}+\frac{1}{x}-x+C$$
$$\Rightarrow \quad \ln|x^2y|=\frac{1}{y}+\frac{1}{x}-x+C \qquad \ldots(i)$$

Given, $y(1)=2$, so we get

$$\Rightarrow \quad \ln\left|1^2\cdot 2\right|=\frac{1}{2}+\frac{1}{1}-1+C$$
$$\Rightarrow \quad \ln|2|=\frac{1}{2}+C\Rightarrow c=\ln|2|-\frac{1}{2}$$

From, Eq. (i), we get

$$\ln|x^2y|=\frac{1}{y}+\frac{1}{x}-x+\ln|2|-\frac{1}{2}$$
$$\Rightarrow \quad \ln|x^2y|-\ln|2|=\frac{1}{y}+\frac{1}{x}-x-\frac{1}{2}$$
$$\Rightarrow \quad \ln\left|\frac{1}{2}x^2y\right|=\frac{1}{y}+\frac{1}{x}-x-\frac{1}{2}$$

Question9

The general solution of the differential equation $\frac{dy}{dx}=\frac{2x+y-3}{2y-x+3}$

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Options:

A.

$x^2 - xy - y^2 + 3x + 3y + c = 0$

B.

$x^2 - xy - y^2 - 3x - 3y + c = 0$

C.

$x^2 + xy - y^2 - 3x - 3y + c = 0$

D.

$x^2 + xy + y^2 + 3x - 3y + c = 0$

Answer: C

Solution:

Given, differential equation

$\frac{dy}{dx} = \frac{2x+y-3}{2y-x+3}$

Let $x = X + h, y = Y + k$, then

$$\begin{aligned}\frac{dY}{dX} &= \frac{2(X+h)+(Y+k)-3}{2(Y+k)-(X+h)+3} \\ &= \frac{2X+Y+2h+k-3}{2Y-X+2k-h+3} \qquad \ldots(i)\end{aligned}$$

Let $2h + k - 3 = 0$ and $-h + 2k + 3 = 0$

Solving these two equations, we get $h = \frac{9}{5}$ and $k = \frac{-3}{5}$

So, Eq. (i) becomes,

$\frac{dY}{dX} = \frac{2X+Y}{2Y-X} \qquad \ldots(ii)$

Let $Y = vX$

$\Rightarrow \frac{dY}{dX} = v + X\frac{dv}{dX'}$

Substituting this into Eq. (ii), we get

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2+v}{2v-1} - v$$

$$\Rightarrow \frac{2+v-2v^2+v}{2v-1}$$

$$= \frac{2v-2v^2+2}{2v-1}$$

$$\Rightarrow \quad \frac{2v-1}{-2v^2+2v+2}dv = \frac{2+v}{2v-1}$$

Integrating both sides, we get

$\int \frac{2v-1}{-2v^2+2v+2}dv = \int \frac{dX}{X}$

Let $u = -2v^2 + 2v + 2$

Then $du = (-4v+2)dv$

$\therefore \frac{-1}{2} \int \frac{du}{u} = \int \frac{dX}{X} \Rightarrow \frac{-1}{2}\ln|u| = \ln|X| + c_1,$

where $c_1 =$ constant

$$\Rightarrow \quad \frac{-1}{2}\ln\left|-2v^2+2v+2\right| = \ln|X| + c_1$$
$$\Rightarrow \quad \ln\left|-2v^2+2v+2\right| = -2\ln|X| + c_2$$

where $c_2 = -2c_1$

$$\Rightarrow \quad \ln\left|X^2\left(-2v^2+2v+2\right)\right| = c_2$$
$$\Rightarrow \quad X^2\left(-2v^2+2v+2\right) = c, \text{ where } c = e^{c_2}$$
$$\Rightarrow \quad X^2\left(\frac{-2Y^2}{X^2} + \frac{2Y}{X} + 2\right) = c$$
$$\Rightarrow \quad \left(-2Y^2 + 2XY + 2X^2\right) = c$$

Substitute X and Y, we get

$$-2\left(y+\frac{3}{5}\right)^2 + 2\left(x-\frac{9}{5}\right)\left(y+\frac{3}{5}\right) + 2\left(x-\frac{9}{5}\right)^2 = c$$

$$\Rightarrow -2\left(y^2+\frac{6}{5}y+\frac{9}{25}\right) + 2\left(xy+\frac{3}{5}x-\frac{9}{5}y-\frac{27}{25}\right) + 2\left(x^2-\frac{18}{5}x+\frac{81}{25}\right) = c$$

$$\Rightarrow \quad -2y^2 - \tfrac{12}{5}y - \tfrac{18}{25} + 2xy + \tfrac{6}{5}x - \tfrac{18}{5}y$$

$$-\frac{54}{25} + 2x^2 - \frac{36}{5}x + \frac{162}{25} = c$$

$$\Rightarrow -2y^2 + 2xy + 2x^2 - \frac{30}{5}y - \frac{30}{5}x + \frac{90}{25} = c$$

$$\Rightarrow \quad 2x^2 + 2xy - 2y^2 - 6x - 6y = c$$

$$\Rightarrow \quad x^2 + xy - y^2 - 3x - 3y = c$$

Thus, the general solution is

$x^2 + xy - y^2 - 3x - 3y + c = 0$

Question10

If $x \log x \frac{dy}{dx} + y = \log x^2$ and $y(e) = 0$, then $y\left(e^2\right) =$

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Options:

A.

0

B.

1

C.

$\frac{1}{2}$

D.

$\frac{3}{2}$

Answer: D

Solution:

Given, $x \log x \frac{dy}{dx} + y = \log x^2$

$$y(e) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{\log x^2}{x \log x}$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2 \log x}{x \log x} = \frac{2}{x}$$

This is a linear differential equation of the form $\frac{dy}{dx} + P(x) y = Q(x)$,

Where $P(x) = \frac{1}{x \log x}$ and $Q(x) = \frac{2}{x}$

Now, IF $= e^{\int \frac{1}{x \log x} dx}$

Let $u = \log x$.

Then $du = \frac{1}{x} dx$

So, IF $= e^{\int \frac{1}{u} du} = e^{\log u} = u = \log x$

So, the solution is given by

$y \cdot \mathrm{IF} = \int Q(x) \cdot \mathrm{IF} dx + c$

$y \log x = \int \frac{2}{x} \log x dx + c$

Let $v = \log x$

Then, $dv = \frac{1}{x} dx$

So, $y \log x = \int 2v dv + c = v^2 + c$

$\Rightarrow y \log x = v^2 + c = (\log x)^2 + c$

Given, $y(e) = 0$

So, $y \log x = (\log x)^2 + c$

$\Rightarrow 0 \cdot \log(e) = (\log e)^2 + c$

$\Rightarrow \quad 0 = 1 + c \Rightarrow c = -1$

$\therefore$ The particular solution is $y \log x = (\log x)^2 - 1$

$\Rightarrow \quad y = \frac{(\log x)^2 - 1}{\log x}$

So, $y\left(e^2\right) = \frac{\left(\log e^2\right)^2 - 1}{\log(e^2)}$

$$= \frac{(2 \log e)^2 - 1}{2 \log(e)} \quad [\because \log_e = 1]$$

$$= \frac{2^2 - 1}{2} = \frac{4-1}{2} = \frac{3}{2}$$

$\therefore y\left(e^2\right) = \frac{3}{2}$

Question11

If the order and degree of the differential equation $x \frac{d^2 y}{dx^2} = \left(1 + \left(\frac{d^2 y}{dx^2}\right)^2\right)^{-1/2}$ are k and l respectively, then k, l are the roots of

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Options:

A.

$x^2 - 5x + 6 = 0$

B.

$x^2 - 3x + 2 = 0$

C.

$x^2 - 7x + 12 = 0$

D.

$x^2 - 6x + 8 = 0$

Answer: D

Solution:

$$x\frac{d^2y}{dx^2} = \left(1 + \left(\frac{d^2y}{dx^2}\right)^2\right)^{\frac{-1}{2}}$$

$$\Rightarrow \quad \left[x\frac{d^2y}{dx^2}\right]^2 = \left[\left(1 + \left(\frac{d^2y}{dx^2}\right)^2\right)^{\frac{-1}{2}}\right]^{-2}$$

$$\Rightarrow \quad \frac{1}{x^2\left(\frac{d^2y}{dx^2}\right)^2} = 1 + \left(\frac{d^2y}{dx^2}\right)^2$$

$$\Rightarrow \quad 1 = x^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2\left(\frac{d^2y}{dx^2}\right)^4$$

$$\Rightarrow \quad x^2\left(\frac{d^2y}{dx^2}\right)^4 + x^2\left(\frac{d^2y}{dx^2}\right)^2 - 1 = 0$$

so, order $= k = 2$

and degree $= l = 4$

Thus, quadratic equation,

$$(x-2)(x-4) = 0$$
$$\Rightarrow \quad x^2 - 6x + 8 = 0$$

Question12

The equation of the curve passing through the point $(0, \pi)$ and satisfying the differential equation $ydx = \left(x + y^3 \cos y\right)dy$ is

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Options:

A.

$x = y^2 \sin y + y \cos^2 y$

B.

$x = y^2 \sin y + 2y \cos^2 \frac{y}{2}$

C.

$x = y^2 \sin y + y \cos^2 \frac{y}{2}$

D.

$x = y^2 \sin y - y \cos^2 y$

Answer: B

Solution:

$$ydx = \left(x + y^3 \cos y\right)dy$$

$$\Rightarrow \quad \frac{dx}{dy} - \frac{x}{y} = y^2 \cos y$$

$$\therefore \quad \text{IF} = e^{\int -\frac{1}{y} dy}$$

$$= e^{\ln(y)^{-1}} = \frac{-1}{y}$$

$$\therefore \quad \frac{1}{y}\left(\frac{dy}{dy}\right) - \frac{x}{y^2} = y \cdot \cos y$$

$$\Rightarrow \quad \int \frac{d}{dy}\left(\frac{x}{y}\right) dy = \int y \cdot \cos y dy$$

$$\Rightarrow \quad \frac{x}{y} = y \sin y - \int \sin y dy$$

$$x = y \sin y + \cos y + C$$

$$\Rightarrow \quad x = y^2 \sin y + y \cos y + cy$$

Put point $(0, \pi)$ into it

$$\Rightarrow \quad 0 = 0 - \pi + c\pi \Rightarrow c = 1$$

$$\therefore \quad x = y^2 \sin y + y \cos y + y$$

$\Rightarrow \quad x = y^2 \sin y + y(1 + \cos y)$

$\Rightarrow \quad x = y^2 \sin y + 2y \cos^2\left(\frac{y}{2}\right)$

Question13

The general solution of the differential equation $(x - (x + y)\log(x + y))dx + xdy = 0$ is

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Options:

A.

$y \log(x + y) = cx$

B.

$\log(x + y) = cy$

C.

$x \log(x + y) = cy$

D.

$\log(x + y) = cx$

Answer: D

Solution:

$(x-(x+y)\log(x+y))dx+xdy=0$
$\Rightarrow \frac{dy}{dx}=\frac{(x+y)\log(x+y)-x}{x}$
$=\frac{(x+y)}{x}\log(x+y)-1$
put. $\quad u=x+y \Rightarrow \frac{du}{dx}=\frac{dy}{dx}+1$
$\Rightarrow \quad \frac{dy}{dx}=\frac{du}{dx}-1$
$\Rightarrow \quad \frac{du}{dx}-1=\frac{u}{x}\log(u)-1$
$\Rightarrow \quad \int \frac{du}{u\log u}=\int \frac{1}{x}dx$
$\Rightarrow \quad \log(\log u)=\log x+\log(c)$
$\Rightarrow \quad \log(\log u)=\log(cx)$
$\Rightarrow \quad \log u=cx$
$\Rightarrow \quad \log(x+y)=cx \qquad (\because u=x+y)$

Question14

The general solution of the differential equation $\sec(x-y+1)dy=dx$ **is**

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Options:

A.

$x+\cot\left(\frac{x-y+1}{2}\right)=C$

B.

$x+\cot(x-y+1)=C$

C.

$x-\cot\left(\frac{x-y+1}{2}\right)=C$

D.

$x-\cot(x-y+1)=C$

Answer: A

Solution:

Given, differential equation is
$\sec(x-y+1)dy = dx$

$\Rightarrow \quad \frac{dy}{dx} = \cos(x-y+1) \qquad \ldots(i)$

Put $x - y = z$

$\Rightarrow 1 - \frac{dy}{dx} = \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = 1 - \frac{dz}{dx}$

So, Eq. (i),

$$\begin{aligned} &\Rightarrow \quad 1 - \frac{dz}{dx} = \cos(z+1) \\ &\Rightarrow \quad \frac{dz}{dx} = 1 - \cos(z+1) = 2\sin^2\left(\frac{z+1}{2}\right) \\ &\Rightarrow \quad dx = \frac{1}{2}\operatorname{cosec}^2\left(\frac{z+1}{2}\right)dz \end{aligned}$$

On integrating,

$x = \frac{1}{2} \times 2\left(-\cot\left(\frac{z+1}{2}\right)\right) + C$

$\Rightarrow \quad x + \cot\left(\frac{x-y+1}{2}\right) = C$

Question15

The differential equation for which $y^2 = 4a(x+a)$ (a is the parameter) is the general solution is

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Options:

A.

$y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$

B.

$y = y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2$

C.

$x = 3\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$

D.

$y = 3x^2\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$

Answer: A

Solution:

Given, $y^2 = 4a(x+a)$ $\quad\quad\quad\quad\ldots(i)$

Differentiate w.r.t x, we get

$$2y\frac{dy}{dx} = 4a(1+0) = 4a$$
$$\Rightarrow \quad y\frac{dy}{dx} = 2a \Rightarrow a = \frac{y}{2}\frac{dy}{dx}$$

From Eq. (1)

$$y^2 = 4 \times \frac{y}{2} \cdot \frac{dy}{dx}\left(x + \frac{y}{2}\frac{dy}{dx}\right)$$
$$\Rightarrow \quad y^2 = 2y\frac{dy}{dx} \times x + y^2\left(\frac{dy}{dx}\right)^2$$
$$\Rightarrow \quad y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$$

Question16

The general solution of the differential equation $\frac{dy}{dx} = \frac{2xy-4x+y-2}{2xy+x-4y-2}$ is

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Options:

A.

$5(y-x) + 2\log\left(\frac{y-2}{x-2}\right) = C$

B.

$2(y-x) - 5\log\left(\frac{y-2}{x-2}\right) = C$

C.

$2(y-x)+5\log\left(\frac{y-2}{x-2}\right)=C$

D.

$5(y-x)-2\log\left(\frac{y-2}{x-2}\right)=C$

Answer: C

Solution:

Given, differential equation is

$$\begin{aligned}
\frac{dy}{dx} &= \frac{2xy-4x+y-2}{2xy+x-4y-2} \\
&= \frac{2x(y-2)+1(y-2)}{2y(x-2)+1(x-2)} \\
&= \frac{(2x+1)(y-2)}{(2y+1)(x-2)} \\
&\Rightarrow \int \frac{2y+1}{y-2}dy = \int \frac{2x+1}{x-2}dx \\
&\Rightarrow \int \frac{2(y-2)+5}{y-2}dy = \int \frac{2(x-2)+5}{x-2}dx \\
&\Rightarrow \int 2dy + \int \frac{5}{y-2}dy = \int 2dx + \int \frac{5}{x-2}dx \\
&\Rightarrow 2y+5\log(y-2) = 2x+5\log(x-2)+c \\
&\Rightarrow 2(y-x)+5\log\left|\frac{y-2}{x-2}\right| = C
\end{aligned}$$

Question17

The differential equation of the family of circles passing through the origin and having centre on X-axis is

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Options:

A.

$(y^2+x^2)dx-2ydy=0$

B.

$\left(y^2-x^2\right) dx-2 x y dy=0$

C.

$\left(y^2-x^2\right) dx+2 y dy=0$

D.

$\left(y^2+x^2\right) dx+2 y dy=0$

Answer: B

Solution:

General equation of circle passing through the origin and having centres on the X-axis.

$$
\begin{aligned}
& x^2+y^2+2 g x=0 \\
\Rightarrow \quad & 2 x+2 y \frac{dy}{dx}+2 g=0 \\
\Rightarrow \quad & g=-x-y \frac{dy}{dx} \\
\Rightarrow \quad & x^2+y^2+2 x\left(-x-y \frac{dy}{dx}\right)=0 \\
\Rightarrow \quad & x^2+y^2-2 x^2-2 x y \frac{dy}{dx}=0 \\
\Rightarrow \quad & -x^2+y^2-2 x y \frac{dy}{dx}=0 \\
\Rightarrow \quad & y^2-x^2=2 x y \frac{dy}{dx} \\
\Rightarrow \quad & \left(y^2-x^2\right) dx-2 x y dy=0
\end{aligned}
$$

Question18

The general solution of the differential equation $\frac{dy}{dx}=\frac{x+y}{x-y}$ is

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Options:

A.

$y-x=c x^2$

B.

$\tan ^{-1}\left(\frac{y}{x}\right)=\log \left(c x \sqrt{x^2+y^2}\right)$

C.

$x+y=cx^2$

D.

$\tan^{-1}\left(\frac{y}{x}\right)=\log\left(c\sqrt{x^2+y^2}\right)$

Answer: D

Solution:

$$\frac{dy}{dx}=\frac{x+y}{x-y}$$

$$\Rightarrow \quad y=vx \Rightarrow v=\frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\Rightarrow \quad v+x\frac{dv}{dx}=\frac{x+vx}{x-vx}$$

$$\Rightarrow \quad v+x\frac{dv}{dx}=\frac{1+v}{1-v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v}{1-v}-v$$

$$\Rightarrow \quad =\frac{1+v-v+v^2}{1-v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v^2}{1-v}$$

$$\Rightarrow \quad \frac{1-v}{1+v^2}dv=\frac{1}{x}dx$$

Integrating both sides, we get

$$\Rightarrow \quad \int\frac{1-v}{1+v^2}dv=\int\frac{1}{x}dx$$

$$\Rightarrow \quad \log x=\int\frac{1}{1+v^2}dv-\int\frac{v}{1+v^2}dv$$

$$\Rightarrow \quad \log x=\tan^{-1}v-\frac{1}{2}\int\frac{1}{t}dt$$

where, $1+v^2=t$

$2vdv=dt$

$vdv=\frac{1}{2}dt$

$\Rightarrow \log x=\tan^{-1} v-\frac{1}{2}\log\left|1+v^2\right|+C$

$\Rightarrow \log x=\tan^{-1}\frac{y}{x}-\frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right|+C$

$\Rightarrow \log x=\tan^{-1}\frac{y}{x}-\frac{1}{2}\log\left|\frac{y^2+x^2}{x^2}\right|+C$

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right)=\log\left(c\sqrt{x^2+y^2}\right)$

Question19

The general solution of the differential equation $\frac{dy}{dx}+\frac{\sec x}{\cos x+\sin x}y=\frac{\cos x}{1+\tan x}$ is

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Options:

A.

$(\cos x+\sin x)y=\sin x+C$

B.

$(\cos x+\sin x)y=\cos x+C$

C.

$(1+\tan x)y=\cos x+C$

D.

$\sec x(\cos x+\sin x)y=\sin x+C$

Answer: D

Solution:

$$\frac{dy}{dx}+\frac{\sec x}{\cos x+\sin x} y=\frac{\cos x}{1+\tan x}$$

$$\text{IF } =e^{\int \frac{\sec x}{\cos x+\sin x} dx}=e^{\int \frac{\sec ^2 x}{1+\tan x} dx}$$

$$=e^{\log (1+\tan x)}=1+\tan x$$

$$\Rightarrow y \cdot(1+\tan x)$$

$$=\int(1+\tan x) \cdot \frac{\cos x}{1+\tan x} dx$$

$$=\int \cos x dx$$

$$=\sin x+C$$

$$\Rightarrow \sec x(\cos x+\sin x) y=\sin x+C$$

Question20

The general solution of the differential equation $\frac{dy}{dx}=\frac{2x^2-xy-y^2}{x^2-y^2}$ is

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Options:

A.

$\log\left|\frac{y^2-2x^2}{x^2}\right|+\sqrt{2}\log\left|\frac{y-\sqrt{2}x}{y+\sqrt{2}x}\right|+2\sqrt{2}\log|x|=C$

B.

$\sqrt{2}\log\left|\frac{y^2-2x^2}{x^2}\right|+\log\left|\frac{y-\sqrt{2}x}{y+\sqrt{2}x}\right|+2\sqrt{2}\log|x|=C$

C.

$\sqrt{2}\log\left|\frac{y^2+2x^2}{x^2}\right|+\log\left|\frac{y+\sqrt{2}x}{y-\sqrt{2}x}\right|+2\sqrt{2}\log|x|=C$

D.

$\log\left|\frac{2x^2-y^2}{x^2}\right|+\sqrt{2}\log\left|\frac{y+\sqrt{2}x}{y-\sqrt{2}x}\right|+\log|x|=C$

Answer: B

Solution:

$$\frac{dy}{dx} = \frac{2x^2 - xy - y^2}{x^2 - y^2}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2x^2 - vx^2 - v^2x^2}{x^2 - v^2x^2}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2 - v - v^2}{1 - v^2} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{2 - v - v^2 - v + v^3}{1 - v^2}$$

$$\Rightarrow \quad \frac{1 - v^2}{v^3 - v^2 - 2v + 2}dv = \frac{1}{x}dx$$

$$\Rightarrow \quad \int \frac{1 - v^2}{(v^2 - 2)(v - 1)}dv = \int \frac{1}{x}dx$$

$$\Rightarrow \quad \int \left(\frac{-v}{v^2 - 2} - \frac{1}{v^2 - 2}\right)dv = \frac{1}{x}dx$$

$$\Rightarrow -\frac{1}{2}\log\left(v^2 - 2\right) - \frac{1}{2\sqrt{2}}\log\frac{v - \sqrt{2}}{v + \sqrt{2}} - \log x + C$$

Put $v = \frac{y}{x}$

$$\Rightarrow \sqrt{2}\log\left|\frac{y^2 - 2x^2}{x^2}\right| + \log\left|\frac{y - \sqrt{2}x}{y + \sqrt{2}x}\right| + 2\sqrt{2}\log|x| = C$$

Question21

If the degree of the differential equation corresponding to the family of curves $y = ax + \frac{1}{a}$ (where $a \neq 0$ is an arbitary constant) is r and it's order is m. Then, the solution of $\frac{dy}{dx} = \frac{y}{2x}, y(1) = \sqrt{r + m}$ is

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Options:

A.

$y = 3^x$

B.

$y^2 = 3x$

C.

$x^2 = 3y$

D.

$y = 3 \log x$

Answer: B

Solution:

$$\begin{aligned}
& y = ax + \frac{1}{a} \\
& \Rightarrow \frac{dy}{dx} = a \Rightarrow \frac{d^2y}{dx^2} = 0 \\
& \Rightarrow x\frac{dy}{dx} + \frac{dx}{dy} = y = 0 \\
& \Rightarrow \quad x\left(\frac{dy}{dx}\right)^2 + 1 - y\frac{dy}{dx} = 0 \\
& \quad r = 2, m = 1 \\
& \text{Now, } \frac{dy}{dx} = \frac{y}{2x}, y(1) = \sqrt{3} \\
& \Rightarrow \int \frac{1}{y} dy = \int \frac{1}{2x} dx \\
& \Rightarrow \log y = \frac{1}{2}\log(x) + \log C \\
& \Rightarrow \log \sqrt{3} = \log C \Rightarrow C = \sqrt{3} \\
& \Rightarrow \log y = \log \sqrt{x} + \log \sqrt{3} \\
& \Rightarrow \quad y = \sqrt{3x} \\
& \therefore \quad y^2 = 3x
\end{aligned}$$

Question22

The general solution of the differential equation $y + \cos x\left(\frac{dy}{dx}\right) - \cos^2 x = 0$ is

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Options:

A.

$(\sec x+\tan x) y=x+\cos x+c$

B.

$(1+\cos x) y=(x+c) \cos x-\cos ^2 x$

C.

$(1+\sin x) y=(x+c) \cos x-\cos ^2 x$

D.

$(\sec x+\tan x) y=x-\sin x+c$

Answer: C

Solution:

$$
\begin{aligned}
& y+\cos x\left(\frac{d y}{d x}\right)-\cos ^2 x=0 \\
\Rightarrow \quad & \frac{d y}{d x}=\frac{\cos ^2 x-y}{\cos x} \\
\Rightarrow \quad & \frac{d y}{d x}+y \sec x=\cos x \\
\Rightarrow \quad & \text { IF }=\int e^{\int \sec x d x}=\sec x+\tan x \\
\Rightarrow \quad & y(\sec x+\tan x)=\int(1+\sin x) d x \\
\Rightarrow \quad & \frac{y(1+\sin x)}{\cos x}=x-\cos x+c \\
\Rightarrow \quad & (1+\sin x) y=(x+c) \cos x-\cos ^2 x
\end{aligned}
$$

Question23

The general solution of the differential equation $\frac{d y}{d x}+x y=4 x-2 y+8$ is

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Options:

A.

$y=4-ce^{-\frac{(x+2)^2}{2}}$

B.

$y=8+ce^{-\frac{x^2}{2}-2x}$

C.

$y=ce^{-(x+2)^2}+x$

D.

$y+2x=ce^{-\frac{x}{2}-2x}$

Answer: A

Solution:

$$\frac{dy}{dx}+xy=4x-2y+8$$
$$\Rightarrow \frac{dy}{dx}+y(x+2)=4x+8$$
$$\text{IF }=e^{\int(x+2)dx}=e^{\frac{(x+2)^2}{2}}$$
$$y\cdot e^{\frac{(x+2)^2}{2}}=\int 4(x+2)e^{\frac{(x+2)^2}{2}}dx$$
$$\text{Put }\frac{(x+2)^2}{2}=t\Rightarrow(x+2)dx=dt$$
$$\therefore y\cdot e^{\frac{(x+2)^2}{2}}=4\int e^t dt+C$$
$$y\cdot e^{\frac{(x+2)^2}{2}}=4e^{\frac{(x+2)^2}{2}}+C$$
$$y=4+Ce^{-\frac{(x+2)^2}{2}}$$

$\because C$ is constant

$\therefore y=4-Ce^{-\frac{(x+2)^2}{2}}$

Question24

The general solution of the differential equation $\left(x+2y^3\right)\frac{dy}{dx}-y=0, y>0$ is

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Options:

A.

$y = x^3 + cy$

B.

$x = y^3 + cy$

C.

$y(1 - xy) = cx$

D.

$x(1 - xy) = cy$

Answer: B

Solution:

$$\left(x + 2y^3\right)\frac{dy}{dx} - y = 0$$
$$\left(x + 2y^3\right)\frac{dy}{dx} = y$$
$$\frac{dx}{dy} = \frac{x + 2y^3}{y}$$
$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$
$$\text{IF} = e^{\int -\frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$$
$$x \cdot \frac{1}{y} = \int 2y^2 \times \frac{1}{y}dy + C$$
$$\Rightarrow \quad \frac{x}{y} = \int 2ydy + C$$
$$\Rightarrow \quad \frac{x}{y} = y^2 + C$$
$$\therefore \quad x = y^3 + Cy$$

Question25

The general solution of the differential equation $\frac{dy}{dx} + \frac{x+y+1}{x-3y+5} = 0$ is

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Options:

A.

$3(y-1)^2 - 2(x+2)(y-1) - (x+2)^2 = C$

B.

$x^2 - 3y^2 - 4xy - 2x - 10y = C$

C.

$3(y+1)^2 + 2(x-2)(y+1) - (x-2)^2 = C$

D.

$x^2 + 3y^2 + 4xy + 2x + 10y = C$

Answer: A

Solution:

Given, $\frac{dy}{dx} + \frac{x+y+1}{x-3y+5} = 0$

$$\Rightarrow xdy - 3ydy + 5dy + xdx + ydx + dx = 0$$
$$\Rightarrow (xdy + ydx) + 5 \cdot dy - 3ydy + xdx + dx = 0$$
$$\Rightarrow d(xy) + 5dy - 3ydy + xdx + dx = 0$$

Now, integrating because every term is variable separable.

$xy + 5y - \frac{3y^2}{2} + \frac{x^2}{2} + x = C$

Clearly, this equation is obtained by solving option

$3(y-1)^2 - 2(x+2)(y-1) - (x+2)^2 = C$

Question26

The differential equation corresponding to the family of parabolas whose axis is along $x = 1$ is

Options:

A.

$\frac{d^2y}{dx^2} - (x-1)\frac{dy}{dx} = 0$

B.

$(x-1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

C.

$\frac{d^2y}{dx^2} + (x-1)\frac{dy}{dx} - y = 0$

D.

$(x-1)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Answer: B

Solution:

Let the equation of parabola having axis $x = 1$ be

$(x-1)^2 = 4ay$

Now, differenting w.r.t. x, we get

$$\begin{aligned} & 2(x-1) = 4ay' \\ \Rightarrow \quad & 2(x-1) = \frac{(x-1)^2}{y} \cdot y' \\ \Rightarrow \quad & 2y = (x-1)y' \end{aligned}$$

Again, differentiating

$$2y' = (x-1)y'' + y'$$
$$\because (x-1)\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

Question27

The general solution of the equation $\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x}e^x$

Options:

A.

$y = xe^x + c$

B.

$y = xe^x + ce^{-x}$

C.

$y = \frac{e^x + c}{x}$

D.

$y = \frac{e^{-x} + cx}{x}$

Answer: C

Solution:

$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

$\text{IF} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Solution of differential equations is

$$y \cdot x = \int e^x \cdot \frac{1}{x} \cdot x dx$$
$$\Rightarrow \quad xy = e^x + c$$
$$\therefore \quad y = \frac{e^x + c}{x}$$

Question28

The general solution of the differential equation

$$\left(x \sin \frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$$

Options:

A.

$\log x+\tan \frac{y}{x}=C$

B.

$\log x+\cos \frac{y}{x}=C$

C.

$\log x-\sin \frac{y}{x}=C$

D.

$\log x-\cos \frac{y}{x}=C$

Answer: D

Solution:

$$
\begin{aligned}
\left(x \sin \frac{y}{x}\right) dy &= \left(y \sin \frac{y}{x}-x\right) dx \\
\frac{dy}{dx} &= \frac{y \sin \frac{y}{x}-x}{x \sin \frac{y}{x}}
\end{aligned}
$$

Put $y=vx$

$$
\begin{aligned}
\frac{dy}{dx} &= v+x \frac{dv}{dx} \\
\Rightarrow v+x \frac{dv}{dx} &= \frac{v \sin v-1}{\sin v} \\
\Rightarrow x \frac{dv}{dx} &= \frac{v \sin v-1-v \sin v}{\sin v} \\
&= -\frac{1}{\sin v} \\
\Rightarrow \quad \int \sin v dv+\int \frac{dx}{x} &= 0 \\
\Rightarrow \quad -\cos v+\ln x &= C \\
\Rightarrow \quad \ln x-\cos \frac{y}{x} &= C
\end{aligned}
$$

Question29

Among the options given below from which option a differential equation of order two can be formed ?

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Options:

A. All circles passing through origin

B. All parabolas passing through origin and having focus on X-axis

C. All the lines passing through the origin

D. All the hyperbolas of the form $x^2 - y^2 = K^2$

Answer: A

Solution:

To determine which option can form a differential equation of order two, consider each geometric configuration:

Option A: All circles passing through the origin

The general equation for a circle passing through the origin is:

$$x^2 + y^2 - 2hx - 2py = 0 \qquad (i)$$

In this equation, there are two arbitrary constants, h and p. The presence of two arbitrary constants implies that the differential equation derived from this circle's equation would be of order two, since each arbitrary constant contributes to the order of the differential equation.

Conclusion

Among the given options, the circles passing through the origin allow for a differential equation of order two. This is due to the presence of two arbitrary constants in the circle's equation, which dictate the order of the corresponding differential equation.

Question30

The differential equation for which $ax + by = 1$ is general solution is

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Options:

A. $\frac{dy}{dx} = x + c$

B. $y\frac{d^2y}{dx^2} + x = 1$

C. $\frac{d^2y}{dx^2} = 0.$

D. $\frac{d^3y}{dx^3} = 0$

Answer: C

Solution:

To find the differential equation that has $ax + by = 1$ as its general solution, follow these steps:

Differentiate the Equation: Start by differentiating both sides of $ax + by = 1$ with respect to x. This gives:

$a + b\frac{dy}{dx} = 0$

Solve for $\frac{dy}{dx}$: From the differentiated equation, solve for $\frac{dy}{dx}$:

$\frac{dy}{dx} = \frac{-a}{b}$

Differentiate Again: Differentiate the expression $\frac{dy}{dx} = \frac{-a}{b}$ with respect to x:

$\frac{d^2y}{dx^2} = 0$

Therefore, the differential equation that has $ax + by = 1$ as its general solution is $\frac{d^2y}{dx^2} = 0$.

Question31

The solution of the differential equation $e^x ydx + e^x dy + xdx = 0$ is

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Options:

A. $e^x + yx^2 = c$

B. $2ye^x + x^2 = c$

C. $ye^x + x^2e^y = c$

D. $e^x + xe^y = c$

Answer: B

Solution:

Given differential equation

$e^x y dx + e^x dy + x dx = 0$
$\Rightarrow (e^x y + x) dx + e^x dy = 0$
$\Rightarrow e^x \frac{dy}{dx} + e^x y + x = 0$
$\Rightarrow \frac{dy}{dx} + y = -\frac{x}{e^x}$

which is linear differential equation

$\therefore \text{IF} = e^{\int P dx} = e^{\int dx} = e^x$

$\therefore$ Required solution

$$
\begin{aligned}
& y e^x = \int e^x \times \left(\frac{-x}{e^x}\right) dx + C \\
\Rightarrow \quad & y e^x = -\frac{x^2}{2} + C \\
\Rightarrow \quad & 2 y e^x + x^2 = C
\end{aligned}
$$

Question32

The differential equation of the family of hyperbols having their centres at origin and their axes along coordinates axes is

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Options:

A. $x y y_2 + x y_1^2 - y y_1 = 0$

B. $x y_2 - x y y_1^2 + y y_1 = 0$

C. $x y y_2 + x y_1^2 + y y_1 = 0$

D. $x y_2 + x y_1^2 - y_1 = 0$

Answer: A

Solution:

The differential equation for a family of hyperbolas with centers at the origin and axes along the coordinate axes is derived from the standard hyperbola equation:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Differentiating this equation with respect to x, we get:

$\frac{2x}{a^2} - \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0$

This simplifies to:

$\Rightarrow \frac{2x}{a^2} = \frac{2y}{b^2} \cdot \frac{dy}{dx} \Rightarrow \frac{y}{x} \cdot \frac{dy}{dx} = \frac{b^2}{a^2}$

Differentiating both sides with respect to x again, we obtain:

$\frac{y}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{x \cdot \frac{dy}{dx} - y \cdot 1}{x^2} = 0$

Which can be rewritten as:

$x \cdot y \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \cdot x - y \cdot \frac{dy}{dx} = 0$

This results in the differential equation:

$xyy_2 + y_1^2 x - yy_1 = 0$

Question33

The general solution of the differential equation $(xy + y^2)dx - (x^2 - 2xy)dy = 0$ is

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Options:

A. $cxy^2 = e^{\frac{x}{y}}$

B. $cxy^2 e^{\frac{x}{y}} = 1$

C. $cxye^{\frac{x}{y}} = 1$

D. $cxy = e^{\frac{x}{y}}$

Answer: B

Solution:

To solve the given differential equation:

$(xy + y^2)\, dx - (x^2 - 2xy)\, dy = 0$

we start by rewriting it in the form of a differential equation:

$\frac{dy}{dx} = \frac{xy+y^2}{x^2-2xy}$

Using the substitution $y = ux$, where u is a function of x, we differentiate y with respect to x:

$\frac{dy}{dx} = u + x\frac{du}{dx}$

Substituting these into the differential equation provides:

$u + x\frac{du}{dx} = \frac{u+u^2}{1-2u}$

Rearranging terms, we get:

$x\frac{du}{dx} = \frac{3u^2}{1-2u}$

This leads to:

$\frac{1-2u}{3u^2}\,du = \frac{dx}{x}$

Integrating both sides yields:

$\int\left(\frac{1}{3u^2} - \frac{2}{3u}\right)du = \int\frac{dx}{x}$

Calculating the integrals gives:

$-\frac{1}{3u} - \frac{2}{3}\log u = \log x + \log c$

Substituting back $u = \frac{y}{x}$:

$-\frac{x}{3y} - \frac{2}{3}\log\frac{y}{x} = \log xc$

After simplification, the general solution is:

$cxy^2e^{\frac{x}{y}} = 1$

Question34

The general solution of the differential equation $(1 + \tan y)(dx - dy) + 2xdy = 0$ is

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Options:

A. $e^x(y\cos x + \sin x) + \sin x = c$

B. $e^x(y\cos x + y\sin x - \sin x) + \cos x = 0$

C. $e^y(x\cos y + x\sin y - \sin y) = c$

D. $e^y(x\cos y + x\sin y + \sin y) = c$

Answer: C

Solution:

To solve the differential equation $(1+\tan y)(dx - dy) + 2x\,dy = 0$, we'll start by rewriting the equation:

$dx + \tan y\,dx - dy - \tan y\,dy + 2x\,dy = 0.$

This simplifies to:

$-dy(1+\tan y - 2x) + dx(1+\tan y) = 0.$

Rewriting the equation, we separate variables:

$\frac{dy}{dx} = \frac{1+\tan y}{1+\tan y - 2x}.$

Alternatively, in terms of $\frac{dx}{dy}$:

$\frac{dx}{dy} = 1 - \frac{2x}{1+\tan y}.$

Now, this can be rewritten as:

$\frac{dx}{dy} + 2\frac{x}{1+\tan y} = 1.$

This is a first-order linear differential equation of the form:

$\frac{dx}{dy} + Px = Q$

where $P = \frac{2}{1+\tan y}$ and $Q = 1.$

To solve it, we calculate the integrating factor (IF):

$\text{IF} = e^{\int P\,dy} = e^{\int \frac{2}{1+\tan y}\,dy}.$

Evaluating the integral, we get:

$\text{IF} = (\sin y + \cos y)e^y.$

The general solution formula is:

$x(\text{IF}) = \int Q \cdot (\text{IF})\,dy + C.$

Substitute the values:

$x(\sin y + \cos y)e^y = \int 1 \cdot (\sin y + \cos y)e^y\,dy + C.$

Splitting the integral:

$= \int \sin y\,e^y\,dy + \int \cos y\,e^y\,dy + C.$

Therefore:

$x(\sin y + \cos y)e^y = \frac{e^y}{2}(\sin y - \cos y) + \frac{e^y}{2}(\cos y + \sin y) + C.$

Simplifying gives:

$x(\sin y + \cos y)e^y = e^y \cdot \sin y + C.$

This simplifies to:

$e^y[x \cos y + x \sin y - \sin y] = C.$

Hence, the general solution is:

$e^y(x \cos y + x \sin y - \sin y) = C.$

Question35

The general solution of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$ is

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Options:

A. $y + \sqrt{x^2 + y^2} = cx^2$

B. $y + \sqrt{x^2 + y^2} = cx$

C. $x + \sqrt{x^2 + y^2} = cy$

D. $x - \sqrt{x^2 + y^2} = cy^2$

Answer: A

Solution:

$$x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$$

$$x\,dy = y\,dx + \sqrt{x^2+y^2}\,dx$$

$$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$$

Put $y = vx \Rightarrow dy = v\,dx + x\,dv$

$$x(v\,dx + x\,dv) = vx\,dx + \sqrt{x^2(1+v^2)}\,dx$$

$$x^2\,dv = x\sqrt{1+v^2}\,dx$$

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

$$\ln\left(v+\sqrt{1+v^2}\right) = \ln x + \ln C$$

$$v + \sqrt{1+v^2} = Cx$$

$$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = Cx$$

$$y + \sqrt{x^2+y^2} = Cx^2$$

Question36

The sum of the order and degree of differential equation

$$x\left(\frac{d^2y}{dx^2}\right)^{1/2} = \left(1+\frac{dy}{dx}\right)^{4/3}$$

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Options:

A. 5

B. 8

C. 12

D. 10

Answer: A

Solution:

$x\left(\frac{d^2y}{dx^2}\right)^{1/2}=\left(1+\frac{dy}{dx}\right)^{4/3}$

Simplifying, on squaring both sides, we get

$$\left(x\frac{d^2y}{dx^2}\right)^{1/2\times 2}=\left(1+\frac{dy}{dx}\right)^{4/3\times 2}$$
$$x^2\left(\frac{d^2y}{dx^2}\right)=\left(1+\frac{dy}{dx}\right)^{8/3}$$

On cubic both sides, we get

$$\left(x^2\frac{d^2y}{dx^2}\right)^3=\left(1+\frac{dy}{dx}\right)^{8/3\times 3}$$
$$x^6\left(\frac{d^2y}{dx^2}\right)^3=\left(1+\frac{dy}{dx}\right)^8$$

Clearly, here the highest derivative with respect to x is 2 , so order is 2 .

Moreover the highest power of highest derivative is 3 i.e. $\left(\frac{d^2y}{dx^2}\right)^3$, so degree is 3 .

$$\begin{aligned}\text{Sum } &= \text{ order } + \text{ degree } = 2+3\\ \text{Sum } &= 5\end{aligned}$$

Question37

The differential equation formed by eliminating arbitrary constants A, B from the equation $y=A\cos 3x+B\sin 3x$ is

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Options:

A. $\frac{d^2y}{dx^2}+y=0$

B. $\frac{d^2y}{dx^2}+9y=0$

C. $\frac{d^2y}{dx^2}-9y=0$

D. $\frac{d^2y}{dx^2}-y=0$

Answer: B

Solution:

We have,

$y = A\cos 3x + B\sin 3x \qquad \dots (i)$

On differentiate y w.r.t x, we get

$$\frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x$$
$$\Rightarrow \frac{d^2y}{dx^2} = -9A\cos 3x - 9B\sin 3x$$
$$\Rightarrow \frac{d^2y}{dx^2} = -9(A\cos 3x + B\sin 3x)$$
$$\Rightarrow \frac{d^2y}{dx^2} = -9y \quad \text{[from Eq. (i)]}$$
$$\Rightarrow \frac{d^2y}{dx^2} + 9y = 0$$

Question38

If $\cos x\frac{dy}{dx} - y\sin x = 6x, \left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right) =$

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Options:

A. $\frac{-\pi^2}{4\sqrt{3}}$

B. $\frac{-\pi^2}{2}$

C. $\frac{-\pi^2}{2\sqrt{3}}$

D. $\frac{\pi^2}{2\sqrt{3}}$

Answer: C

Solution:

We start with the given differential equation:

$\cos x\frac{dy}{dx} - y\sin x = 6x, \left(0 < x < \frac{\pi}{2}\right)$

and the initial condition:

$y\left(\frac{\pi}{3}\right) = 0$

Rewriting the differential equation:

$\frac{dy}{dx} - y\frac{\sin x}{\cos x} = \frac{6x}{\cos x}$

This simplifies to:

$\frac{dy}{dx} - y\tan x = 6x\sec x$

This is in the form of the linear differential equation:

$\frac{dy}{dx} + Ay = B$

where $A = -\tan x$ and $B = 6x\sec x$.

The integrating factor (IF) is found using:

$\text{IF} = e^{\int \text{Adx}} = e^{-\int \tan x\, dx} = e^{-\log\sec x} = \frac{1}{\sec x}$

Applying the integrating factor, the solution becomes:

$y \cdot \frac{1}{\sec x} = \int 6x\sec x \cdot \frac{1}{\sec x}\, dx$

Simplifying, we get:

$\frac{y}{\sec x} = 3x^2 + C$

where C is a constant. Using the initial condition $y\left(\frac{\pi}{3}\right) = 0$:

$0 = 3\left(\frac{\pi}{3}\right)^2 + C \implies C = -\frac{\pi^2}{3}$

Thus, substituting back, we have:

$\frac{y}{\sec x} = 3x^2 - \frac{\pi^2}{3}$

Solving at $x = \frac{\pi}{6}$:

$y = \sec x \cdot \left(3x^2 - \frac{\pi^2}{3}\right)$

Calculating:

$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$3\left(\frac{\pi}{6}\right)^2 = \frac{\pi^2}{12}$

Therefore,

$y = \frac{2}{\sqrt{3}} \cdot \left(\frac{\pi^2}{12} - \frac{\pi^2}{3}\right) = \frac{2}{\sqrt{3}} \cdot \left(-\frac{\pi^2}{4}\right)$

Simplifying gives:

$y\left(\frac{\pi}{6}\right) = \frac{-\pi^2}{2\sqrt{3}}$

Question39

$\frac{dy}{dx}=\frac{y+x\tan\frac{y}{x}}{x}\Rightarrow\sin\frac{y}{x}=$

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Options:

A.

cx^2

B.

cx

C.

cx^3

D.

cx^4

Answer: B

Solution:

We have,

$\frac{dy}{dx}=\frac{y+x\tan\left(\frac{y}{x}\right)}{x}$

Put $y=vx$

$\Rightarrow\quad \frac{d}{dx}\cdot(vx)=\frac{vx+x\tan\left(\frac{vx}{x}\right)}{x}$

$\Rightarrow\quad v+x\frac{dv}{dx}=\frac{x(v+\tan v)}{x}$

$\Rightarrow\quad \frac{xdv}{dx}=v+\tan v-v$

$\Rightarrow\quad x\frac{dv}{dx}=\tan v\Rightarrow\frac{dv}{\tan v}=\frac{dv}{x}$

$\Rightarrow\quad \frac{\cos vdv}{\sin v}=\frac{dx}{x}$

On taking integration both sides, we get

$$\Rightarrow \quad \int \frac{\cos v dv}{\sin v} = \int \frac{dx}{x}$$

Let $\quad \sin v = u$

$\cos v dv = du$

$$\Rightarrow \quad \int \frac{1}{u} du = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log u = \log x + \log C$$

$$\Rightarrow \quad \log(\sin v) = \log x + \log C$$

$$\Rightarrow \quad \log\left(\frac{\sin v}{x}\right) = \log C \Rightarrow \frac{\sin v}{x} = C$$

$$\Rightarrow \quad \sin v = xC$$

$$\Rightarrow \quad \sin \tfrac{y}{x} = xC$$

$$\Rightarrow \quad v^2 = 1.44$$

$$\Rightarrow \quad v = 1.2 \text{ m/s}$$

Question40

The differential equation formed by eliminating a and b from the equation $y = ae^{2x} + bxe^{2x}$ is

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Options:

A. $y'' - 4y' - 4y = 0$

B. $y'' + 4y' - 4y = 0$

C. $y'' + 4y' + 4y = 0$

D. $y'' - 4y' + 4y = 0$

Answer: D

Solution:

To find the differential equation by eliminating the parameters a and b from the equation $y = ae^{2x} + bxe^{2x}$, we take the derivatives with respect to x and express them in terms of y.

First Derivative:

$y' = \frac{d}{dx}(ae^{2x} + bxe^{2x}) = 2ae^{2x} + (2bxe^{2x} + be^{2x})$

$= 2ae^{2x} + 2bxe^{2x} + be^{2x} = 2y + be^{2x}$

Second Derivative:

$y'' = \frac{d}{dx}(2ae^{2x} + 2bxe^{2x} + be^{2x})$

$= 4ae^{2x} + 2be^{2x}(2x+1) + b(2e^{2x})$

$= 4ae^{2x} + 4bxe^{2x} + 2be^{2x} + 2be^{2x}$

$= 4ae^{2x} + 4bxe^{2x} + 4be^{2x}$

$= 4(ae^{2x} + bxe^{2x}) + 4be^{2x}$

$= 4y + 4be^{2x}$

Constructing the Differential Equation:

Substitute into the expression $y'' - 4y' + 4y$:

$y'' - 4y' + 4y = [4y + 4be^{2x}] - 4[2y + be^{2x}] + 4y$

$= 4y + 4be^{2x} - 8y - 4be^{2x} + 4y$

$= 0$

The expression simplifies to zero, confirming that the differential equation $y'' - 4y' + 4y = 0$ is satisfied, supporting the correctness of option (D).

Question41

If $y = a^3 e^{y^2 x + c}$ is the general solution of a differential equation, where a and c are arbitrary constants and b is fixed constant, then the order of differential equation is

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Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Given the general solution $y = a^3 e^{b^2 x + c}$, where a and c are arbitrary constants and b is a fixed constant, we are tasked with identifying the order of the differential equation related to this solution.

To find the order, let's derive the expression for y:

$$\begin{aligned} y' &= \frac{d}{dx}\left(a^3 e^{b^2 x + c}\right) \\ &= a^3 b^2 e^{b^2 x + c}. \end{aligned}$$

Notice that we can express this in terms of y:

$$y' = b^2 y.$$

This differential equation, $y' - b^2 y = 0$, is a first-order linear differential equation. The order of a differential equation is determined by the highest derivative present. In this case, since the highest derivative is y', the order is 1.

Question42

The solution of differential equation $\left(x + 2y^3\right)\frac{dy}{dx} = y$ Is

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Options:

A. $x = y(2xy + c)$

B. $x = y\left(y^2 + c\right)$

C. $y = x\left(x^2 + 0\right)$

D. $xy = \frac{y^4}{2} + c$

Answer: B

Solution:

To solve the differential equation $(x + 2y^3)\frac{dy}{dx} = y$, we begin by rearranging terms:

$$\frac{x+2y^3}{y} = \frac{dx}{dy}$$

This can be rewritten as:

$$\frac{dx}{dy} = \frac{x}{y} + 2y^2$$

This gives us a form where:

$\frac{dx}{dy} - \frac{x}{y} = 2y^2$

We recognize this as a linear first-order differential equation, which can be solved using an integrating factor. The integrating factor (IF) is derived as follows:

$\text{IF} = e^{\int -\frac{1}{y}\,dy} = e^{-\ln|y|} = \frac{1}{y}$

Apply the integrating factor to both sides of the differential equation:

$\frac{x}{y} = \int \left(2y^2 \cdot \frac{1}{y}\right) dy$

This integral becomes:

$\frac{x}{y} = 2\int y\,dy$

Calculate the integral:

$\frac{x}{y} = 2\left(\frac{y^2}{2}\right) + C$

Simplifying gives:

$\frac{x}{y} = y^2 + C$

Therefore, multiplying both sides by y results in:

$x = y(y^2 + C)$

Thus, the solution to the differential equation is:

$x = y(y^2 + C)$

Question43

Order and degree of the differential equation $\frac{d^3y}{dx^3} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{2}}$, respectively are

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Options:

A. 5,2

B. 3,5

C. 3,2

D. 2,3

Answer: C

Solution:

Given differential equation is

$$\frac{d^3y}{dx^3} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{2}}$$

$$\Rightarrow \quad \left(\frac{d^3y}{dx^3}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^5$$

Highest derivative is third order and power of highest order derivative is 2 .

Hence, order is 3 and degree is 2 .

Question44

Integrating factor of the differential equation $\sin x \frac{dy}{dx} - y \cos x = 1$ is

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Options:

A. $\sin x$

B. $\cos x$

C. $\sec x$

D. $\operatorname{cosec} x$

Answer: D

Solution:

To find the integrating factor of the given differential equation, we start with the equation:

$\sin x \frac{dy}{dx} - y \cos x = 1$

Rewriting the equation, we have:

$\frac{dy}{dx} - \frac{\cos x}{\sin x} y = \frac{1}{\sin x}$

This simplifies to:

$\frac{dy}{dx} - \cot x \cdot y = \operatorname{cosec} x$

The standard form for integrating factors is:

$IF = e^{\int -\cot x\, dx}$

Calculating the integral, we find:

$IF = e^{-\log|\sin x|}$

This can further be expressed as:

$IF = e^{\log(\sin x)^{-1}} = \frac{1}{\sin x}$

Therefore, the integrating factor for the given differential equation is:

$\frac{1}{\sin x} = \operatorname{cosec} x$

Question45

The general solution of the differential equation $\left(x \sin \frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$ is

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Options:

A. $\cos \frac{x}{y} = \log_6 x + c$

B. $\cos \frac{x}{y} = \log_e y + c$

C. $\cos \frac{y}{x} = \log_e x + c$

D. $\cos \frac{y}{x} = \log_e y + c$

Answer: C

Solution:

To solve the given differential equation:

$\left(x \sin \frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$

we start by expressing it in differential form:

$\frac{dy}{dx} = \frac{y \sin \frac{y}{x} - x}{x \sin \frac{y}{x}}$

Introduce a new variable v such that $y = vx$. This implies:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute $y = vx$ into the differential equation:

$v + x\frac{dv}{dx} = \frac{vx\sin v - x}{x\sin v}$

Simplify:

$x\frac{dv}{dx} = \frac{vx\sin v - x - vx\sin v}{x\sin v} = \frac{-x}{x\sin v} = \frac{-1}{\sin v}$

This results in the following separable equation:

$\sin v\, dv = -\frac{dx}{x}$

Integrate both sides:

$\int \sin v\, dv = -\int \frac{dx}{x}$

The integration yields:

$-\cos v = -\log x + C$

Rearranging gives:

$\log x - \cos v = C$

Substituting back $v = \frac{y}{x}$, we arrive at the solution:

$\cos\left(\frac{y}{x}\right) = \log_e x + C$

Question46

The sum of the order and degree of the differential equation $\frac{d^4y}{dx^4} = \left\{c + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}$ is

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Options:

A. 4

B. 6

C. 5

D. 8

Answer: B

Solution:

To determine the sum of the order and degree of the given differential equation:

$$\frac{d^4y}{dx^4} = \left\{C + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}$$

First, rewrite the equation in a form where the degree is clear:

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left(C + \left(\frac{dy}{dx}\right)^2\right)^3$$

From this, we observe:

The order of the differential equation is the highest derivative present, which is 4.

The degree of the differential equation is the power of the highest derivative when the equation is free of fractions and any radical signs. Here, the degree is 2 (as the highest derivative, $\frac{d^4y}{dx^4}$, is squared).

Therefore, the sum of the order and degree is $4 + 2 = 6$.

Question47

The general solution of the differential equation $(x + y)ydx + (y - x)xdy = 0$ is

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Options:

A. $x + y\log(cy) = 0$

B. $\frac{y}{x} = \log(xy) + c$

C. $x + y\log(cxy) = 0$

D. $\frac{y}{x} = \log(cxy)$

Answer: C

Solution:

To solve the differential equation:

$(x+y)y\,dx + (y-x)x\,dy = 0$

we can start by rewriting the equation:

$\frac{dy}{dx} = \frac{(x+y)y}{(x-y)x}$

Substitute $y = vx$, where v is a function of x. Then, the derivative $\frac{dy}{dx}$ becomes:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute $y = vx$ into the differential equation:

$v + x\frac{dv}{dx} = \frac{(x+vx)vx}{(x-vx)x}$

Simplify the equation:

$v + x\frac{dv}{dx} = \frac{v+v^2}{1-v}$

Rearranging gives:

$x\frac{dv}{dx} = \frac{v+v^2-v+v^2}{1-v}$

Next, separate variables and integrate:

$\int\left(\frac{1-v}{v^2}\right)dv = \int 2x\,dx$

This can be rewritten as:

$\int\left(\frac{1}{v^2} - \frac{1}{v}\right)dv = \int 2x\,dx$

Integrate both sides:

$-\frac{1}{v} - \log v = 2\log x + \log c$

Substitute back $v = \frac{y}{x}$:

$-\frac{x}{y} = \log\frac{y}{x} + \log x^2 + \log c$

Which simplifies to the equation:

$-\frac{x}{y} = \log(cxy)$

Thus, the general solution is:

$x + y\log(cxy) = 0$

Question48

The general solution of the differential equation $(y^2 + x + 1)dy = (y+1)dx$ is

Options:

A. $x+2+(y+1)\log(y+1)^2=y+c$

B. $x+2+\log(y+1)^2=\frac{y}{y+1}+c$

C. $\frac{x}{y+1}=\log(y+1)^2+y+c$

D. $\frac{x+2}{y+1}+\log(y+1)^2=y+c$

Answer: D

Solution:

We are given the differential equation:

$(y^2+x+1)\,dy=(y+1)\,dx$

This can be rewritten as:

$\frac{dx}{dy}-\frac{1}{y+1}x=\frac{y^2+1}{y+1}$

This is a linear differential equation. To solve it, we first find the integrating factor (IF):

$\mathrm{IF}=e^{\int P\,dy}=e^{\int\frac{-1}{y+1}\,dy}=e^{-\log(y+1)}=\frac{1}{y+1}$

Multiplying through by the integrating factor gives:

$x\cdot\frac{1}{y+1}=\int\frac{1}{y+1}\cdot\frac{y^2+1}{y+1}\,dy=\int\frac{y^2+1}{y^2+2y+1}\,dy$

This simplifies as:

$=\int\left(1-\frac{2y}{y^2+2y+1}\right)dy+C$

Carrying out the integration:

$=y-\int\left[\frac{2y+2}{y^2+2y+1}-\frac{2}{(y+1)^2}\right]dy+C$

$=y-\log(y+1)^2+2\int\frac{1}{(y+1)^2}\,dy+C$

$=y-\log(y+1)^2-\frac{2}{y+1}+C$

Thus, the general solution is:

$\frac{x+2}{y+1}+\log(y+1)^2=y+C$

Question49

The difference of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{-\frac{7}{2}}\left(\frac{d^3y}{dx^3}\right)^2-\left(\frac{d^2y}{dx^2}\right)^{-\frac{5}{2}}\left(\frac{d^4y}{dx^4}\right)=0$ is

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Options:

A. 5

B. 3

C. 4

D. 2

Answer: D

Solution:

We have,

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{-7}{2}}\left(\frac{d^3y}{dx^3}\right)^2-\left(\frac{d^2y}{dx^2}\right)^{\frac{-5}{2}}\left(\frac{d^4y}{dx^4}\right)=0$$

$$\Rightarrow\left(\frac{d^3y}{dx^3}\right)^2\left(\frac{d^2y}{dx^2}\right)^{\frac{5}{2}}=\left(\frac{d^4y}{dx^4}\right)\left(\frac{d^2y}{dx^2}\right)^{\frac{7}{2}}$$

On squaring both sides, we get

$$\Rightarrow\left(\frac{d^3y}{dx^3}\right)^4\left(\frac{d^2y}{dx^2}\right)^5=\left(\frac{d^4y}{dx^4}\right)^2\left(\frac{d^2y}{dx^2}\right)^7$$

$\therefore$ Order is 4 and degree is 2 .

Hence, required difference $=2$

Question50

If $xdy+\left(y+y^2x\right)dx=0$ and $y=1$ at $x=1$, then

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Options:

A. $y=\frac{x}{1+\log x}$

B. $y=\frac{1+\log x}{x}$

C. $y=x(1+\log x)$

D. $y=\frac{1}{x(1+\log x)}$

Answer: D

Solution:

To solve the given differential equation:

$x\, dy+\left(y+y^2 x\right) dx=0$

Rearrange it as:

$\frac{dy}{dx}+\frac{y}{x}+y^2=0$

Let's make a substitution to simplify this equation. Set $\frac{1}{y}=t$, which implies $-\frac{1}{y^2} \frac{dy}{dx}=\frac{dt}{dx}$.

This transforms the equation into:

$\frac{dt}{dx}-\frac{1}{x} t=1$

This is a first-order linear differential equation of the form:

$\frac{dt}{dx}-\frac{1}{x} t=1$

To solve this, we use an integrating factor (IF):

$\mathrm{IF}=e^{\int-\frac{1}{x}\, dx}=\frac{1}{x}$

Multiplying through by the integrating factor gives:

$\frac{1}{x} t=\int \frac{1}{x}\, dx+C$

Integrating the right side:

$\frac{1}{x} t=\log x+C$

Substituting back in terms of y:

$\frac{1}{xy}=\log x+C$

Given the initial condition $y=1$ when $x=1$:

$\frac{1}{1 \cdot 1}=\log 1+C \Rightarrow 1=0+C \Rightarrow C=1$

So the equation becomes:

$\frac{1}{xy}=\log x+1$

Rearranging, we find:

$xy(1 + \log x) = 1$

Finally, solving for y:

$y = \frac{1}{x(1+\log x)}$

Question51

The solution of $xdy - ydx = \sqrt{x^2 + y^2}dx$ when $y(\sqrt{3}) = 1$ is

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Options:

A. $y^2 + \sqrt{x^2 + y^2} = x^2$

B. $5y - \sqrt{x^2 + y^2} = x^2$

C. $y + \sqrt{x^2 + y^2} = x^2$

D. $5y^2 - \sqrt{x^2 + y^2} = x$

Answer: C

Solution:

To solve the differential equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$, with the initial condition $y(\sqrt{3}) = 1$, follow these steps:

Start by rewriting the equation:

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Apply the substitution $y = vx$, where v is a function of x. Then, differentiate to get:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute back into the equation:

$v + x\frac{dv}{dx} = \frac{\sqrt{x^2+(vx)^2}+vx}{x}$

Simplify:

$v + x\frac{dv}{dx} = \frac{\sqrt{x^2(1+v^2)}+vx}{x}$

$x\frac{dv}{dx}=\sqrt{1+v^2}$

Separate the variables:

$\frac{1}{\sqrt{1+v^2}}dv=\frac{1}{x}dx$

Integrate both sides:

$\log\left|v+\sqrt{1+v^2}\right|=\log x+\log c$

Combine the logs:

$\left|v+\sqrt{1+v^2}\right|=cx$

Substituting back for $v=\frac{y}{x}$:

$\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}=cx$

$y+\sqrt{x^2+y^2}=cx^2$

Use the initial condition $y(\sqrt{3})=1$:

$1+\sqrt{(\sqrt{3})^2+1^2}=c(\sqrt{3})^2$

$1+2=3c$

$3=3c\implies c=1$

The required solution is:

$y+\sqrt{x^2+y^2}=x^2$

Question52

The differential equation representing the family of circles having their centres of Y -axis is $\left(y_1=\frac{dy}{dx}\text{ and }y_2=\frac{d^2y}{dx^2}\right)$

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Options:

A. $y_2=y\left(y_1^2+1\right)$

B. $y_2=xy\left(y_1^2+1\right)$

C. $x_2=y_1\left(y_1^2+1\right)$

D. $xy_2=y\left(y_1^2+1\right)$

Answer: C

Solution:

To find the differential equation representing a family of circles with centers on the Y-axis, let's consider the circle with center $(0, K)$ and radius r. The general equation for such a circle is:

$x^2 + (y - K)^2 = r^2 \quad \ldots$(i)

Differentiate equation (i) with respect to x:

$2x + 2(y - K)\frac{dy}{dx} = 0$

Which simplifies to:

$y - K = \frac{-x}{\frac{dy}{dx}} \quad$ or $\quad K = y + \frac{x}{\frac{dy}{dx}}$

Let's denote $\frac{dy}{dx}$ as y_1. So, from the above, we have:

$y - K = \frac{-x}{y_1} \quad \Rightarrow \quad K = y + \frac{x}{y_1}$

Now, differentiate this equation with respect to x again:

$0 = 1 + (y - K)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$

Substituting $y - K = \frac{-x}{y_1}$ into this, we get:

$1 + \left(-\frac{x}{y_1}\right)\frac{d^2y}{dx^2} + y_1^2 = 0$

This can be rearranged to:

$1 - \frac{x\frac{d^2y}{dx^2}}{y_1} + y_1^2 = 0$

$x\frac{d^2y}{dx^2} = y_1(y_1^2 + 1)$

So, the differential equation is:

$x\frac{d^2y}{dx^2} = y_1(y_1^2 + 1)$

Question53

The general solution of the differential equation $(\sin y \cos^2 y - x \sec^2 y)dy = (\tan y)dr$, is

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Options:

A. $\tan y = 3x\cos^3 y + c$

B. $x(\sec y + \tan y) = \cos^2 y + c$

C. $y\sin y = x^2\cos^2 y + c$

D. $3x\tan y + \cos^3 y = c$

Answer: D

Solution:

To solve the given differential equation:

$(\sin y\cos^2 y - x\sec^2 y)dy = (\tan y)dx,$

we start by rewriting it in the standard linear form:

$\frac{dx}{dy} + \frac{x}{\sin y\cos y} = \cos^3 y.$

Recognizing this as a linear differential equation in x, we determine the integrating factor (IF):

Compute the Integrating Factor (IF):

$\text{IF} = e^{\int \frac{1}{\sin y\cos y}\,dy}.$

Solve the Integral:

$\int \frac{1}{\sin y\cos y}\,dy = 2\int \csc 2y\,dy.$

Solving this integral gives:

$e^{\int \csc 2y\,dy} = e^{\log|\csc 2y - \cot 2y|} = \csc 2y - \cot 2y.$

Thus, the integrating factor simplifies to:

$\text{IF} = \frac{1-\cos 2y}{\sin 2y} = \frac{2\sin^2 y}{2\sin y\cos y} = \tan y.$

Apply the Integrating Factor:

Multiply the entire linear differential equation by this integrating factor:

$x \times \tan y = \int \cos^3 y \times \tan y\,dy.$

Integrate the Right-Hand Side:

Substitute $\cos^2 y = 1 - \sin^2 y$. Then:

$\int \cos^3 y\tan y\,dy = \int \cos^2 y\sin y\,dy.$

Perform the integration:

$= -\frac{\cos^3 y}{3} + C.$

Resulting General Solution:

Therefore, the general solution to the differential equation is:

$3x \tan y + \cos^3 y = C.$

Question54

The general solution of the differential equation $(x - y - 1)dy = (x + y + 1)dx$ is

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Options:

A. $\tan^{-1}\left(\frac{y+1}{x}\right) - \frac{1}{2}\log\left(x^2 + y^2 + 2y + 1\right) = 0$

B. $(x - y) + \log(x + y) = c$

C. $y^2 - x^2 + xy - 3y - x = c$

D. $(x - y - 1)^2(x + y + 1)^3 = c$

Answer: A

Solution:

To solve the given differential equation, we start by analyzing the equation:

$(x - y - 1)dy = (x + y + 1)dx$

This can be rewritten in differential form as:

$\frac{dy}{dx} = \frac{x+y+1}{x-y-1}$

This expression can also be represented as:

$\frac{dy}{dx} = \frac{x+(y+1)}{x-(y+1)}$

Introduce a substitution where $y + 1 = z$. Thus, we have:

$\frac{dy}{dx} = \frac{dz}{dx}$

After substitution, the equation becomes:

$\frac{dz}{dx} = \frac{x+z}{x-z}$

Next, we use another substitution $z = vx$. Therefore, we differentiate z with respect to x:

$\frac{dz}{dx} = v + x\frac{dv}{dx}$

Substituting this back into our equation results in:

$v + x\frac{dv}{dx} = \frac{1+v}{1-v}$

Reorganize terms and separate variables:

$x\frac{dv}{dx} = \frac{1+v-v+v^2}{1-v}$

This simplifies to:

$\left(\frac{1-v}{1+v^2}\right)dv = \frac{dx}{x}$

Integrating both sides, we have:

$\int\left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv = \int\frac{dx}{x}$

The integrals yield:

$\tan^{-1}v - \frac{1}{2}\log\left|1+v^2\right| = \log x + C$

Substitute back $v = \frac{z}{x}$:

$\tan^{-1}\frac{z}{x} - \frac{1}{2}\log\left|\frac{z^2}{x^2}\right| = \log x + C$

Simplifying using $z = y + 1$:

$\tan^{-1}\left(\frac{y+1}{x}\right) - \frac{1}{2}\log\left|\frac{x^2+(y+1)^2}{x^2}\right| - \log x = C$

Finally, this simplifies to:

$\tan^{-1}\left(\frac{y+1}{x}\right) - \frac{1}{2}\log\left|x^2+y^2+2y+1\right| = C$

Question55

The general solution of the differential equation $\frac{dy}{dx} = \cos^2(3x+y)$ is $\tan^{-1}\left(\frac{\sqrt{3}}{2}\tan(3x+y)\right) = f(x)$. Then, $f(x) =$

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Options:

A. $2\sqrt{3}(x+C)$

B. $x+C$

C. $\frac{x+C}{2\sqrt{3}}$

D. $\frac{\sqrt{3}}{2}(x+C)$

Answer: A

Solution:

Here, $\frac{dy}{dx}=\cos^2(3x+y)$

On putting $3x+y=t$

$$3+\frac{dy}{dx}=\frac{dt}{dx}$$
$$\Rightarrow \quad \frac{dy}{dx}=\frac{dt}{dx}-3 \Rightarrow \frac{dt}{dx}-3=\cos^2 t$$
$$\Rightarrow \quad \frac{dt}{dx}=\cos^2 t+3 \Rightarrow \frac{dt}{\cos^2 t+3}=dx$$

Integrate both side,

$$\int \frac{dt}{\cos^2 t+3}=\int dx$$

$$\Rightarrow \int \frac{\sec^2 t dt}{1+3\sec^2 t}=\int dx \Rightarrow \int \frac{\sec^2 t}{1+3+3\tan^2 t}=\int dx$$
$$\Rightarrow \int \frac{\sec^2 t dt}{4+3\tan^2 t}=\int dx$$

On putting tant $=m$,

$$\sec^2 t dx=dm=\frac{1}{4}\int \frac{dm}{1+\left(\frac{\sqrt{3}}{2}m\right)^2}x+C$$

$$=\frac{1}{4}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{2}m\right)=x+C$$

$$=\frac{1}{2\sqrt{3}}\tan^{-1}\left[\frac{\sqrt{3}}{2}\tan t\right]=x+C \quad [\because m=\tan t]$$

$$=\tan^{-1}\left[\frac{\sqrt{3}}{2}\tan(3x+y)=2\sqrt{3}(x+C)\right] \quad [\therefore t=3x+y]$$

So, $f(x)=2\sqrt{3}(x+C)$

Question56

If the general solution of the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x$ is $y=\tan x-1+Ce^{-\tan x}$ satisfies $y\left(\frac{\pi}{4}\right)=1$,

then $C =$

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Options:

A. e

B. 1

C. -1

D. $\frac{1}{e}$

Answer: A

Solution:

Given,

$$\begin{aligned} & \cos^2 x \frac{dy}{dx} + y = \tan x \\ \Rightarrow & \frac{dy}{dx} + y \sec^2 x = \tan x \cdot \sec^2 x \quad \ldots \text{(i)} \end{aligned}$$

Here, $p = \sec^2 x$

$$\Rightarrow \quad \int p dp = \int \sec^2 x dx = \tan x$$

$IF = e^{\tan x}$

Multiplying Eq. (i) by IF, we get

$e^{\tan x} \frac{dy}{dx} + e^{\tan x} y \sec^2 x = e^{\tan x} \cdot \tan x \cdot \sec^2 x$

Integrating both sides, we get

$y e^{\tan x} = \int e^{\tan x} \tan x \cdot \sec^2 x dx$

On putting $\tan x = t$,

$$\begin{aligned} & \sec^2 x dx = dt \\ \therefore \quad & y e^t = \int t e' dt = e^t (t-1) + C \\ \therefore \quad & y e^{\tan x} = \tan x - 1 + C e^{-\tan x} \\ \text{If} \quad & y\left(\frac{\pi}{4}\right) = 1 \\ \Rightarrow \quad & 1 = 1 - 1 + C e^{-1} \Rightarrow 1 = C e^{-1} \\ \therefore \quad & C = e \end{aligned}$$

Question57

Assertion (A) Order of the differential equations of a family of circles with constant radius is two.

Reason (R) An algebraic equation having two arbitrary constants is general solution of a second order differential equation.

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Options:

A. A and R are true, R is the correct explanation to A

B. A is true, R is false

C. A and R are true, R is not the correct explanation to A

D. A is false, R is true

Answer: A

Solution:

Any circle with given radius can be written as, $(x-h)^2+(y-k)^2=a^2$ where (h, k) be the centre of the circle which is variable. So, in above algebraic equation, there are two arbitrary constant h and k. Hence, order of differential equation will be second order.Hence, assertion and reason are true and reason is the correct explanation to assertion.

Question58

If l and m are order and degree of a differential equation of all the straight lines at constant distance of P units from the origin, then $lm^2+l^2m=$

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Options:

A. 2

B. 6

C. 12

D. 30

Answer: B

Solution:

Any straight line which is at a constant distance p from the origin is

$x \cos \alpha + y \sin \alpha = p \quad .... \text{(i)}$

Differentiating Eq. (i) on both sides w.r.t. x,

$\cos \alpha + \sin \alpha \frac{dy}{dx} = 0 \Rightarrow \tan \alpha = -\frac{dx}{dy} = -\frac{1}{y'}$

Now, $\sin \alpha = \frac{1}{\sqrt{1+y'^2}}, \cos \alpha = \frac{-y'}{\sqrt{1+y'^2}}$

On putting Eq. (i), we get

$\frac{x(-y')}{\sqrt{1+y'^2}} + \frac{y}{\sqrt{1+y'^2}} = p$

$\Rightarrow \left(y - xy'^2\right)^2 = p^2 \left(l + y'^2\right)$
Order $(l) = 1$
Degree $(m) = 2$
$\therefore lm^2 + l^2 m = (1 \cdot 2)^2 + (1)^2 \cdot 2 = 4 + 2 = 6$

Question59

If $2x - y + C \log(|x - 2y - 4|) = k$ is the general solution of $\frac{dy}{dx} = \frac{2x-4y-5}{x-2y+2}$, then $C =$

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Options:

A. 4

B. 2

C. 3

D. -4

Answer: C

Solution:

Given, $\frac{dy}{dx}=\frac{2x-4y-5}{x-2y+2}$

Let $x-2=v$

$\therefore 1-2\frac{dy}{dx}=\frac{dv}{dx} \Rightarrow \frac{dy}{dx}=\frac{\left(1-\frac{dv}{dx}\right)}{2}$

$\therefore \frac{\left(1-\frac{dv}{dx}\right)}{2}=\frac{2v-5}{v+2}$

$$\Rightarrow 1-\frac{dv}{dx}=\frac{4v-10}{v+2} \Rightarrow 1-\frac{4v-10}{v+2}=\frac{dv}{dx}$$
$$\Rightarrow \frac{dv}{dx}=\frac{12-3v}{v+2} \Rightarrow \int \frac{(v+2)}{4-v}dv=3\int dx+C$$
$$\Rightarrow -1\int \frac{-v+4-6}{(4-v)}dv=3x+C$$
$$\Rightarrow -\int dv+6\int \frac{dv}{4-v}=3x+C$$

$$\Rightarrow -(v)-6\log|4-v|=3x+C$$
$$\Rightarrow -(x-2y)-6\log|4-x+2y|=3x+C$$
$$\Rightarrow (4x-2y)+6\log|4-x+2y|=-C$$
$$\Rightarrow (2x-y)+3\log|x-2y-4|=k \quad \left(k=-\frac{C}{2}\right)$$

Thus, C = 3

Question60

By eliminating the arbitrary constants from $y=(a+b)\sin(x+c)-de^{x+e+f}$, then differential equation has order of

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Options:

A. 6

B. 4

C. 3

D. 5

Answer: C

Solution:

Given, $y=(a+b)\sin(x+c)-de^{x+e+f}$ ... (i)

$\frac{dy}{dx}=(a+b)\cos(x+c)-de^{x+c+f}$ ... (ii)

$\frac{d^2y}{dx^2}=-(a+b)\sin(x+c)-de^{x+e+f}$ ... (iii)

$=\frac{d^3y}{dx^3}=-(a+b)\cos(x+c)-de^{x+c+f}$ ... (iv)

On adding Eqs. (i) and (iii), we get

$\left(y+\frac{d^2y}{dx^2}\right)=-2de^{x+e+f}$ ... (v)

Adding Eqs (ii) and (iv), we get

$\left(\frac{dy}{dx}+\frac{d^3y}{dx^3}\right)=-2de^{x+e+s}$ ... (vi)

On equating Eqs. (v) and (vi) we get

$$\frac{d^3y}{dx^3}+\frac{dy}{dx}=\frac{d^2y}{dx^2}+y$$
$$\Rightarrow \frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$$

Order of above differential equation is 3.

Question61

If the solution of $\frac{dy}{dx}-y\log_e 0.5=0, y(0)=1$, and $y(x)\to k$, as $x\to\infty$, then $k=$

Options:

A. ∞

B. -1

C. 1

D. 0

Answer: D

Solution:

$$\frac{dy}{dx} - y\log_c 0.5 = 0$$
$$\Rightarrow \frac{dy}{y} = \log 0.5 dx$$
$$\Rightarrow \quad \int \frac{dy}{y} = \int \log 0.5 dx$$
$$\Rightarrow \log y = (\log 0.5)x + C \quad \text{... (i)}$$
$$\because \quad y(0) = 1 \text{ i.e at } x = 0, y = 1$$
$$\therefore \log(1) = 0 + C$$
$$\Rightarrow \quad C = 0$$

Therefore, Eq. (i) becomes $\log y = (\log 0.5)x$

$$\Rightarrow \quad \log y = \log(0.5)^x$$
$$\Rightarrow \quad y = (0.5)^x$$

When $x \to \infty, y \to k$

$$\therefore \quad k = (0.9)^\infty$$
$$\Rightarrow \quad k = \left(\frac{1}{2}\right)^\infty = \frac{1}{2^\infty} = \frac{1}{\infty} = 0$$

Question62

$y = Ae^x + Be^{-2x}$ satisfies which of the following differential equations?

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Options:

A. $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$

B. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

C. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

D. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

Answer: D

Solution:

$y = Ae^x + Be^{-2x}$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = Ae^x - 2Be^{-2x}$ ... (i)

Again on differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = Ae^x + 4Be^{-2x}$ .... (ii)

Adding Eqs. (ii) and (iii), we get

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 2Ae^x + 2Be^{-2x} = 2y$$
$$\Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

Question63

If $y = \sin(\sin x)$ and $y'' + f(x) \cdot y' + g(x) \cdot y = 0$, then $f(x) \cdot g(x)$ is equal to

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Options:

A. $\frac{1}{2}\sin(2x)$

B. $\frac{1}{2}\cos(2x)$

C. $\sin(2x)$

D. $\cos(2x)$

Answer: A

Solution:

$$y=\sin(\sin x),\text{ then } y'=\cos(\sin x)\cdot\cos x$$
$$y''=-\sin x\cos(\sin x)-\cos^2 x\sin(\sin x)$$
$$=-\sin x\cos(\sin x)-(\cos^2 x)y$$
$$=-\sin x\left(\frac{y'}{\cos x}\right)-(\cos^2 x)y$$
$$\Rightarrow y''+(\tan x)y'+(\cos^2 x)y=0$$
$$\Rightarrow \quad f(x)=\tan x, g(x)\cos^2 x$$
$$\Rightarrow f(x)\cdot g(x)=\sin x\cos x=\frac{1}{2}(2\sin x\cos x)$$
$$f(x)\cdot g(x)=\frac{1}{2}\sin 2x$$

Question64

The equation of the curve passing through the point $\left(0,\frac{\pi}{4}\right)$ and satisfying the differential equation $(e^x\tan y)dx+(1+e^x)\sec^2 y)dy=0$ is given by

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Options:

A. $(1+e^x)\tan y=2$

B. $1+e^x=2\tan y$

C. $1+e^x=2\sec y$

D. $(1+e^x)\tan y=k$

Answer: A

Solution:

$(e^x \tan y)dx + (1+e^x)\sec^2 y)dy = 0$ ...... (i)

Its of the form $Mdx + Ndy = 0$

Now, $\frac{\partial M}{\partial y} = e^x \sec^2 y$ and $\frac{\partial N}{\partial x} = e^x \sec^2 y$

$\Rightarrow$ Eq. (i) is exact.

Thus, its solution will be

$\int Mdx + \int Ndy = C$

(treat y-constant) No terms of x included

$$\int e^x \tan y dx + \int \sec^2 y dy = C$$
$$e^x \tan y + \tan y = C$$
$$\tan y\,(1+e^x) = C \quad \text{......(ii)}$$

Since, Eq. (ii) passes through $(0, \pi/4)$

$\tan \frac{\pi}{4}(1+1) = C \Rightarrow C = 2$

From Eq. (ii), $\tan y\,(1+e^x) = 2$

Question65

The solution of the differential equation $2x\left(\frac{dy}{dx}\right) - y = 4$ represents a family of

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Options:

A. ellipse

B. parabola

C. straight line

D. circle

Answer: B

Solution:

$$2x\frac{dy}{dx} - y = 4$$

$$\frac{dy}{dx} - \left(\frac{y}{2x}\right) = \left(\frac{4}{2x}\right)$$

$$\text{IF} = e^{\int \frac{-1}{2x} dx} = e^{\frac{\log x}{-2}} = \frac{1}{\sqrt{x}}$$

$$\left(\frac{1}{\sqrt{x}} y\right) = \int \frac{2}{x\sqrt{x}} dx \Rightarrow \frac{y}{\sqrt{x}} = 2\left(\frac{x^{-\frac{3}{2}+1}}{\frac{-3}{2}+1}\right) + C$$

$$\frac{y}{\sqrt{x}} = 2\left(\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right) + C$$

$$y = C\sqrt{x} - 4 \Rightarrow (y+4)^2 = C^2 x$$

$\Rightarrow$ Represents a parabola.

Question66

The solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ is

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Options:

A. $y = 3\sin x + 4\cos x$

B. $y = x^2$

C. $y = x + 2$

D. $y = \log x$

Answer: A

Solution:

Given, differential equation $\frac{d^2y}{dx^2} + y = 0$

A solution is function of x which satisfies given differential equation.

Option (a) $y = 3\sin x + 4\cos x$

$$\frac{dy}{dx} = 3\cos x - 4\sin x$$

$$\frac{d^2y}{dx^2} = -3\sin x - 4\cos x = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$
